

Inflation as a Solution to the Early Universe Entropy Problem?

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There exists the ‘entropy problem’ of the early universe, that is, why did the universe begin with an extremely low entropy and how did it evolve into such high entropy at late times? It has been long believed that inflation cannot be the solution since it requires an extremely low entropy to ever occur. However, we point out that since the inflation is always accompanied with a horizon, the correct probability of inflation is associated with the quantum entanglement entropy, which should in principle be larger than what considered previously. This motivates us to reexamine the issue by computing the evolution of the cosmological entanglement entropy in the early universe. We invoke a toy model of nonlinear generalized Chaplygin gas (GCG), which has the advantage of providing a smooth and unitary transition between the inflation epoch and the radiation dominant era. We found that soon after the onset of the inflation, the total entanglement entropy rapidly decreases to a minimum, and it rises monotonically afterwards throughout the remainder of the inflation and the radiation epochs. This indicates that the universe does not need to begin with an extremely low entropy; its smallness can be naturally induced by the dynamics of inflation itself. We believe that our computation largely captures the essential feature of entropy evolution and can provide us insights beyond the toy model.

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I. INTRODUCTION

An increasing amount of observational evidence supports the notion that the early universe has undergone an epoch of inflation. However we are still far from understanding the underlying assumptions and resolving some of its most crucial issues. One of the open questions about inflation is the entropy problem[1][2][3][4]: why did the universe start from an extremely low entropy initial state that in turn can later give rise to the arrow of time in accordance with the second law of thermodynamics?

Thermodynamic arrow tells us that a system will evolve towards the increase of entropy, which simply states that a system is more likely to evolve from a state with low occurrence probability to that with a higher probability. Since the universe is old, in order to explain the thermodynamic arrow of time the entropy in the early universe must therefore be extremely small, which requires a special initial condition.

On the other hand, the standard inflation paradigm assumes the initial condition of the universe to be realized generically. This conflict [4] becomes more acute if we regard the initial entropy as the probability for spontaneous

formation of a homogeneous domain in the inflationary universe. An extremely low initial entropy required by the thermodynamic arrow then suggests that the initial universe only has a very small probability to evolve into the current universe through inflation.

Several solutions have been suggested in the literature. One approach is the spontaneous eternal inflation [3][5], where inflation is assumed to occur both forward and backward in time. Another is the so-called bubble cosmology[6], where universes were induced as a realization of string landscape[7][8][9].

In this paper, we seek a solution to the entropy problem within the present framework of the standard model of cosmology without relying on assumptions like symmetric time arrows or string theory. The framework will only be effective, in that new physics is expected to occur near the initial singularity predicted by General Relativity, and detailed mechanism for the transition from inflation to a radiation-dominated universe is still not well-understood.

Our aim is to show that a low entropy state can be dynamically produced from inflation itself, followed by the generation of a large amount of entropy. This possibility has long been refuted in the previous literature [10][11][12][13] based on the argument that inflation itself requires an extremely special initial condition to occur. However, we hold the point of view that whether the initial condition is special or not depends largely on how one measures the probability of the degrees of freedom.

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For example, in the recent development of loop quantum cosmology [14] it was found rather differently that the probability for inflation to happen is much higher using their measure. We should like to point out an important fact, which was overlooked by those previous authors, that *inflation is accompanied with a horizon*. Therefore the correct measure of the probability for inflation should be given by quantum *entanglement entropy*. Since the presence of the horizon increases the entanglement entropy by the entanglement between the observable and the unobservable regions, inflation should have a higher probability to occur. This insight motivates us to seek new dynamical solution to the entropy problem within the inflation paradigm.

We notice that in the literature there are examples of decreasing entanglement entropy [15][16] due to the situation where the system is out of equilibrium or simply from long-range quantum correlation. In particular, a decrease of tripartite entropy after inflation is shown in [16], which inspires us to consider the evolution of bipartite entropy from quantum entanglement. Indeed, in a recent effort by two of the present authors (PC & YN) [17], it was found that a minimum of the entanglement entropy induced from the entanglement between cosmological perturbations is reached at an early stage of the inflation. The calculation made there, however, was classical. Since we expect quantum effects to be prominent in the beginning of the universe, each degree of freedom must subject to quantum fluctuations. We therefore re-address the issue properly in this paper.

This paper is organized as follows. In section II we briefly summarize the reasons and methods for considering entanglement entropy as entropy budget of early universe, which consist of homogeneous and inhomogeneous parts. In section III we introduce the effective model used in our calculation. Detailed treatment and numerical results are summarized in next two sections IV and V, which together consist of the total contribution to entanglement entropy. To make comparison with the previous result in [17], the computation therein is reproduced in section V for completeness. Results in each section are discussed, and the main conclusion is summarized in the last section.

II. ENTANGLEMENT ENTROPY AS ENTROPY BUDGET

In the present framework, most conventional considerations of entropy are thermal, statistical and entanglement entropy accounted by von Neumann. Since early universe experienced accelerated expansion, it is far out of equilibrium, and thermal entropy is ill-defined for lack of a universal temperature. Statistical mechanics is also not directly applicable on present situation due to long-range nature of gravitational force.

On the other hand, von Neumann entropy from quantum entanglement is perfectly well-defined in non-

equilibrium system, and can cope with problems statistical entropy encounters in gravitational systems like loss of information. Moreover, von Neumann entropy fits into unitary framework of entire universe in that it measures information encoded in entanglement between subsystems, while thermal entropy is of dissipation nature. Since the early universe can be described as a quantum system without strong decoherence, it is justifiable to invoke von Neumann entropy to find the evolution of total entropy of the universe. We will thus use the quantum entanglement entropy instead of thermal entropy in the following discussion to help clarify our setup and to quantify the amount of information in a quantum system.

Here we will clarify what we mean by von Neumann entanglement entropy. Entanglement entropy of von Neumann measures the degree of bipartite entanglement, and thus depends on partitions of the system. Multipartite entanglement entropy thus seems necessary in order to characterize total entanglement entropy of a system; however, the method is difficult to applied on general systems [18]. We suggest one reasonable way to quantify the total entanglement entropy of a quantum system is to sum up von Neumann entanglement entropies of all bi-partite subsystems. Since we only concern the entropy change in the evolution, we can subtract a ground value to regularize any potential infinity. More on regularization will be discussed in later sections.

To track entropy evolution, a specified setting is needed, but yet general enough to characterize the essential features. The early universe we are investigating at roughly can be divided into two stages: inflationary, and radiation dominated eras. From the requirement of unitarity, our model must provide smooth transition between those stages; in addition, we will work in traditional FRW homogeneous background. An candidate with minimal ad hoc assumptions for our mathematical model is generalized Chaplygin gas (GCG), which can realize the transition smoothly in its equation of state. Instead of attaching it as real substance, we only regard it as an effective description and convenient way to track down entropy evolution. The details of our model, including parameter space, are discussed in section (III A). The result from this model, however, we believe to be quite general; see section (VI).

In the construction of entanglement entropy evolution, there are two contributions arose from two distinctive ways of dividing the universe. Regarding the entire universe as unitary quantum mechanical system consist of observable and unobservable regions, von Neumann entropy then measures the quantum entanglement between those two parts of universe due to the evolution of background metric, and is thus a homogeneous contribution. In this unitary setting von Neumann entropy characterizes the probability of different histories, since the total amount of information is kept constant by Louville's theorem. The entanglement entropy is then obtained by the standard Wheeler-DeWitt formalism, where configurations of fields represent different histories in the path

integral sense.

The second contribution comes from the entanglement between two opposite momentum sectors of the universe, and is entitled inhomogeneous entanglement entropy. In our model of homogeneous background, it attributes to entanglement of pairwise matter momentum modes, which can be computed by standard cosmological perturbation. The total inhomogeneous entanglement entropy is then obtained by summing over the entire momentum spectrum.

III. GENERALIZED CHAPLYGIN GAS

A. The Model

When applied to cosmology, the Chaplygin gas[19] models the change of cosmic content by regulating the equation of state of the background fluid instead of the form of the potential. It was first suggested by Kamenshchik[20] in an attempt to smoothly interpolate the de Sitter phase and the radiation dominant era without *ad hoc* assumptions. Generalized Chaplygin gas models (GCG) were subsequently suggested[21][22][23], some of which managed to unify dark energy and dark matter[24]. Instead of endowing it as a physical substance, we will only invoke it as an effective description.

The fluid density of GCG is given by[19]:

$$\rho = \left(\frac{A}{a^{1+\beta}} + \frac{B}{a^{4(1+\gamma)}} \right)^{\frac{1}{1+\gamma}}, \quad (1)$$

where A and B are positive constants. Note that in this expression we have implicitly taken a reference scale factor at some point during inflation to be $a_0 = 1$. One can also manifest this reference scale by rescaling $A \mapsto Aa_0^{1+\beta}$, $B \mapsto Ba_0^{4(1+\gamma)}$.

Consider the parameters $\beta = -1$ and $\gamma = -2$. Observe that when taking the limit $a \rightarrow 0$, one recovers from Eq.(1) the fluid density of de Sitter space characterizing the inflation era; when taking $a \rightarrow \infty$, the fluid density becomes that of a gas in the radiation-dominated era. Thus the two eras can be smoothly patched under this effective description. The constraints on the parameters β and γ to realize this scenario will be discussed in Section III B.

From the conservation of energy one can deduce the pressure for the Chaplygin gas as

$$p = \frac{1}{3}\rho + \frac{1+\beta-4(1+\gamma)}{3(1+\gamma)} \left(\rho - \frac{B}{a^{4(1+\gamma)}} \rho^{-\gamma} \right). \quad (2)$$

After eliminating the scale factor a , one can obtain the equation of state for generalized Chaplygin gas.

One can also model GCG with an underlying minimal coupling scalar field ϕ by matching the energy momentum with the fluid described above. Denoting the scalar

potential by $V(\phi)$, we have:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p = \frac{1}{2}\dot{\phi}^2 - V(\phi). \quad (3)$$

The classical expression for the scalar potential as well as the equation of motion can be immediately obtained by relating a and ϕ , as in [19]. For this purpose, we only need the limiting case for the early-time and the late-time approximations, respectively, with the partition roughly coincides with the crossing between the inflation and the radiation-dominated epochs. These are

- Early time [$a/a_0 \ll (A/B)^{1/\delta}$]:

$$\phi(a) \sim \phi_0 + \frac{1}{\kappa} \sqrt{\frac{1+\beta}{1+\gamma}} \ln \frac{a}{a_0}, \quad (4)$$

$$V(\phi) \sim \left[1 - \frac{1+\beta}{6(1+\gamma)} \right] A^{1/(1+\gamma)} a_0^{(\lambda/\kappa)^2} e^{-\lambda(\phi-\phi_0)}, \quad (5)$$

where $\lambda \equiv \kappa \sqrt{(1+\beta)/(1+\gamma)} > 0$ is the coupling constant, $\phi_0 = \phi(a_0)$, and $\kappa^2 \equiv 8\pi G$.

- Late time [$a/a_0 \gg (A/B)^{1/\delta}$]:

$$\phi \sim \phi'_0 + \frac{2}{\kappa} \ln \frac{a}{a_0}, \quad (6)$$

$$V(\phi) \sim \frac{1}{3} B^{1/(1+\gamma)} a_0^4 e^{-2\kappa(\phi-\phi'_0)}. \quad (7)$$

B. Parameter Space of GCG

In this subsection we will briefly summarize the constraints on the generalized Chaplygin gas (1)

First we require that the inflation takes place before radiation dominant era, which amounts to the condition:

$$\frac{B}{a_0^{4(1+\gamma)}} \ll \frac{A}{a_0^{1+\beta}} \quad (8)$$

as $a \rightarrow 0$. Since both A and B are positive, we have

$$\delta \equiv 1 + \beta - 4(1 + \gamma) > 0. \quad (9)$$

Two more conditions should be imposed on β and γ so as to insure that our model does interpolate between an early inflationary phase of the type of quintessence and a subsequent radiation dominant phase. These are: (i) the energy density must induce a period of inflation and (ii) the inflation should not cause a super-inflationary expansion; i.e. $0 < \dot{H}$. That is, a super-accelerating phase of the universe, where the energy density grows as the universe expands, should not occur.

By combining these two ansatz with the above inequality (9), we arrive at the following additional constraints:

$$\begin{aligned} 1 + \beta &< 0, \\ 1 + \gamma &< 0, \\ 1 + \beta - 2(1 + \gamma) &> 0. \end{aligned} \quad (10)$$

In particular, the last constraint results from the requirement for an inflationary period $\ddot{a} > 0$ as $a \rightarrow 0$, which directly leads to the constraint on the coupling constant of the GCG scalar field (5),

$$\lambda < \sqrt{2}\kappa. \quad (11)$$

It is possible to have inflation at intermediate value of a (see Appendix A). However, in this paper we will only consider the case of inflation at the very beginning.

C. Quantizing Generalized Chaplygin Gas

Before entering into explicit calculations, we like to briefly comment on the problem one often encounters when implementing the quantization on the generalized Chaplygin gas model, such as the one we introduced in Section III A. When promoting a scalar field ϕ to a quantum field, one expects that the scalar potential would receive loop corrections. By inspecting on Eq. (5), one can quickly recognize that this model is nonrenormalizable by simply noticing the scalar coupling λ has mass dimension -1 . However, since the scalar field ϕ serves as the inflaton in the inflationary period, which decoheres entirely sometime after the onset of inflation, we do not expect the already decohered field to regain quantum fluctuations in the remaining period of inflation. We can therefore safely invoke the semiclassical approximation to the system beyond the point of decoherence, where one only has to consider the finite saddle-point contribution, and no regularization is required. On the other hand, before the onset of decoherence of the inflaton, one does have to treat the scalar field ϕ that represents the Chaplygin gas fully quantum mechanically, thus some sort of regularization is still needed.

In our computation of the homogeneous part of the entropy, we invoke the minisuperspace approximation, which amounts to truncating large degrees of freedom and keeping only the homogeneous modes. Therefore the minisuperspace approximation serves as our regularization, based on which a finite result is obtained in Section IV D. For the correct evaluation of the inhomogeneous part of the entropy at early times, one does have to impose a cutoff scale, below which our model is effective. We will discuss more on the issue of quantum corrections to the inhomogeneous entropy at early times at Appendix B.

IV. ENTANGLEMENT ENTROPY: HOMOGENEOUS PART

As explained in section II, the contribution to the cosmological entanglement entropy is divisible into two parts; in this section we will address the homogeneous contribution using the Wheeler-DeWitt (WDW) formalism and the minisuperspace treatment. The wavefunction of the universe depends on two modes: one from

gravitation, the other from the scalar field of the Chaplygin gas with the spatial dependence suppressed, thus the name “homogeneous”.

We will treat the gravitational degree of freedom using the flat FRW metric:

$$ds^2 = -N^2 dt^2 + a(t)^2 d\vec{x}^2. \quad (12)$$

Throughout this paper we will follow the convention $\kappa^2 \equiv 8\pi G = 6$.

The action of our model is $S = S_{\text{EH}} + S_{\text{GCG}}$, where the Einstein-Hilbert action can be written as

$$S_{\text{EH}} = - \int d^3\vec{x} dt N \frac{1}{2} a^{-1} P_a^2, \quad (13)$$

where $P_a = \dot{a}a/N$ is the canonical momentum conjugated with a . The spatial integral gives a volume v_0 , which should not concern us and will be dropped hereafter. The matter action is

$$S_{\text{GCG}} = \int d^3\vec{x} dt N a^3 \left(\frac{1}{2N} \dot{\phi}^2 - V(\phi) \right), \quad (14)$$

where $V(\phi)$ is the scalar potential of the Chaplygin gas.

Varying with the lapse function N , one can obtain the Hamiltonian equation $H = 0$, which can be quantized straightforwardly using the canonical prescription. Hereafter we will stick to the gauge $N = 1$.

A. Semiclassical Calculation of Entanglement Entropy

As mentioned in Section III C, since the scalar field ϕ decoheres sometime during inflation, we can substitute ϕ by a using the equation of motion. Therefore, the matter action is replaced essentially by the fluid density, and the Hamiltonian equation boils down to

$$H = -\frac{1}{2} a^{-1} P_a^2 + a^3 \rho(a) = 0, \quad (15)$$

which, after the canonical quantization $P_a \mapsto -i\frac{\partial}{\partial a}$, becomes

$$\hat{H}\Psi = \left[\hat{\mathcal{O}}_{\text{Grav}} + \mathcal{O}_{\text{Matter}} \right] \Psi = 0, \quad (16)$$

where $\Psi = \Psi(a)$ is the wavefunction of the universe, $\hat{}$ denotes differential operators, and $\mathcal{O}_{\text{Matter}}$ equals essentially to the generalized Chaplygin gas density (1). The equation above is the Wheeler-DeWitt equation with one dynamical degree of freedom a . For later convenience, we will factor out a a^{-1} factor in both terms, making it resemble the Schrödinger equation for a unit mass particle.

Since we would like to compute the von Neumann entropy from the matter wavefunction, which can be extracted from the Born approximation [25]

$$\Psi \approx e^{iS_{\text{EH}}} \psi, \quad (17)$$

where $e^{iS_{EH}}$ is the gravitational part, which, to the leading order, satisfies the Hamilton-Jacobi equation

$$\left(\frac{dS_{EH}}{da}\right)^2 - 2a^4\rho(a) = 0.$$

The matter wavefunction ψ satisfies the Schrödinger equation corresponding to the potential of the scalar field:

$$V(\phi(a)) = V(a). \quad (18)$$

To simplify the computation, we will invoke the early-time and the late-time approximations, i. e., Eq.(5) and Eq.(7), respectively. In particular, the potential has the asymptotic forms:

$$V(a) \sim V_0 (a/a_0)^{-(1+\beta)/(1+\gamma)}, \quad \text{Early-time} \quad (19)$$

$$V(a) \sim V'_0 (a/a_0)^{-4}, \quad \text{Late-time} \quad (20)$$

with V_0 and V'_0 as well as the criterion of division given in Section III A. We observe that the potential is positive. Therefore the stationary ground state wavefunction decays exponentially.

Since this is a semiclassical treatment, we are only allowed to obtain the wave function through the WKB approximation:

$$\psi(a) \sim C e^{-\int_{a_i}^a \sqrt{2V(x)} dx}, \quad (21)$$

where a_i is a point in time before the semiclassical approximation breaks down. (cf. Section IV B)

From the above equation we can compute the von Neumann entropy with the density matrix $\bar{\rho}(a_1, a_2) = |\psi(a_1)|^2 \delta(a_1 - a_2)$,

$$\Delta S(a) = - \int_{a_i}^a \bar{\rho} \ln \bar{\rho} da'. \quad (22)$$

A bar is used to distinguish it with the fluid density.

The numerical results are shown in Figure 1 for early times and Figure 2 for late times, where the late time is entirely in the radiation-dominated epoch.

From the figures one can observe that the evolution of the “homogeneous entropy” beyond the gravitational decoherence point is roughly increasing followed by decreasing in radiation epoch. This is expected since particle horizon only present in inflationary period, thus entanglement entropy is expected to decrease after some time in the transition.

B. Breakdown of Semiclassical Treatment

As has been pointed out, the semiclassical treatment of Chaplygin gas, as used in most literature, breaks down at the very beginning of the universe. Physically, we expect that at the onset of inflation the inflaton should execute large quantum mechanical fluctuations that seed

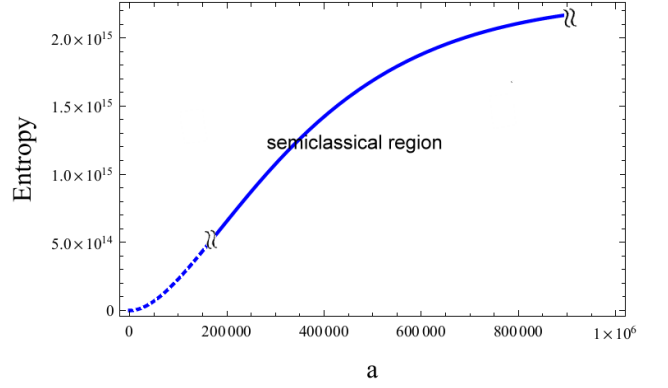


FIG. 1: The evolution of the homogeneous entanglement entropy under the early-time approximation, Eq.(4) & Eq.(5), contributed from the entanglement between the observable universe and that outside the horizon, obtained from the WKB wavefunction of Wheeler-DeWitt equation Eq.(21).

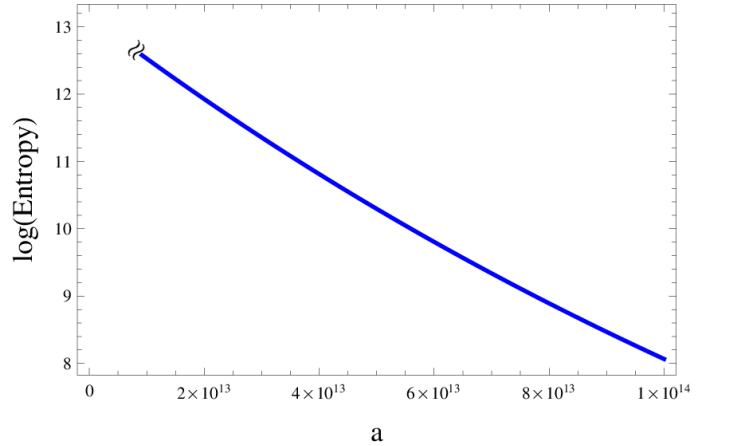


FIG. 2: The evolution of the homogeneous entanglement entropy under the late-time approximation, Eq.(6) & Eq.(7), contributed from the entanglement between the observable universe and that outside the horizon, within the radiation dominant era

the CMB spectrum. One should therefore treat the scalar field quantum mechanically before the decoherence of inflation occurs.

Consider the scalar potential after substituting the saddle point relation, $\phi = \phi(a)$, which at early times is of the form $V(a) \sim a^{-(1+\beta)/(1+\gamma)}$. The validity of the semiclassical (WKB) approximation requires the quantum potential to vary sufficiently slowly. To be more precise, it requires the WKB characteristic length

$$l_{\text{WKB}} \sim \frac{\hbar}{p_{cl}} = \frac{\hbar}{\sqrt{2V}} \quad (23)$$

to change slowly over the distance, that is,

$$\frac{dl_{\text{WKB}}}{da} \ll 1, \quad (24)$$

where without loss of generality we have taken the mass to be 1.

When inserting the early time potential in Eq.(18), we find

$$\left| \frac{d l_{\text{WKB}}}{da} \right| \sim \left| \frac{d(V^{-1/2})}{da} \right| \sim a^{-\frac{1+\beta-2(1+\gamma)}{2(-1-\gamma)}}. \quad (25)$$

However, with the constraints on parameters in Eq.(10), it is easy to see that the right hand side goes to infinity when $a \rightarrow 0$, thus violating the validity condition of WKB semiclassical approximation. A full quantum treatment of both a and ϕ is therefore necessary for early-time universe, which we will carry out in the following Section IV C.

C. Wavefunction at the Very Beginning of Universe

We will implement a fully quantum mechanical treatment at early times via the full Wheeler-DeWitt equation:

$$[\hat{\mathcal{O}}_{\text{Grav}} + \hat{\mathcal{O}}_{\text{Matter}}] \Psi(a, \phi) = 0, \quad (26)$$

where we have promoted ϕ to be an independent operator. Written explicitly, we have (in the convention $\kappa^2 = 6$)

$$\left(\frac{1}{2} a \frac{\partial}{\partial a} a \frac{\partial}{\partial a} - \frac{1}{2} \frac{\partial^2}{\partial \phi^2} + a^6 V(\phi) \right) \Psi(a, \phi) = 0, \quad (27)$$

or, with $\alpha = \ln a$,

$$\left(\frac{1}{2} \frac{\partial^2}{\partial \alpha^2} - \frac{1}{2} \frac{\partial^2}{\partial \phi^2} + e^{6\alpha} V(\phi) \right) \Psi(\alpha, \phi) = 0. \quad (28)$$

In order to recover the Chaplygin gas density at the saddle point, we adopt the scalar potential $V(\phi) = V_0 e^{\lambda \phi}$ from Eq.(6). Thus the WDW equation is just the scalar field with an exponential potential in the FRW metric. This model is known to be exactly solvable using the following change of variables[26]:

$$x = \frac{\sqrt{2V_0}}{3} \frac{e^{3\alpha - \lambda\phi/2}}{1 - (\lambda/6)^2} \left[\cosh \theta(a, \phi) + \frac{\lambda}{6} \sinh \theta(a, \phi) \right], \quad (29)$$

$$y = \frac{\sqrt{2V_0}}{3} \frac{e^{3\alpha - \lambda\phi/2}}{1 - (\lambda/6)^2} \left[\sinh \theta(a, \phi) + \frac{\lambda}{6} \cosh \theta(a, \phi) \right], \quad (30)$$

where $\theta(a, \phi) \equiv 3\phi - \lambda\alpha/2$. The WDW equation Eq.(28) becomes

$$\left[\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + 1 \right] \Psi(x, y) = 0, \quad (31)$$

which can be easily solved to give

$$\Psi = \int dk \left[C_1(k) e^{i(kx - \sqrt{k^2 - 1}y)} + C_2(k) e^{-i(kx - \sqrt{k^2 - 1}y)} \right]. \quad (32)$$

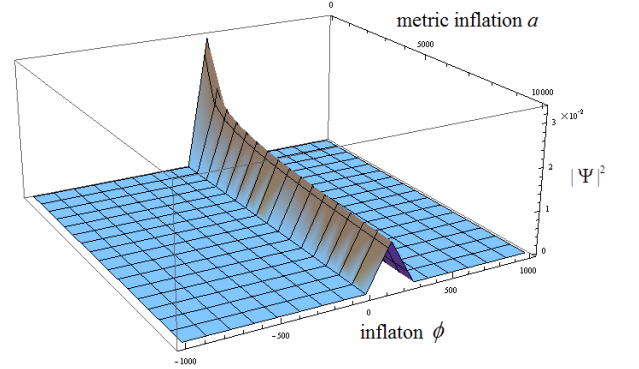
Take a Gaussian wave packet data with the peak at k_0 and width σ , we obtain a wave packet solution

$$|\Psi|^2 = \frac{2}{\sqrt{\pi}} \frac{(k_0^2 - 1)^{3/2} \sigma}{\sqrt{(k_0^2 - 1)^3 + \sigma^4 y^2}} \times \exp \left[- (k_0^2 - 1)^2 \sigma^2 \frac{(\sqrt{k_0^2 - 1}x - k_0 y)^2}{(k_0^2 - 1)^3 + y^2 \sigma^4} \right]. \quad (33)$$

A typical profile of $|\Psi|^2$ is shown in Figure 3. We can see that the distribution is peaked at the saddle point with a nonzero quantum-mechanical dispersion similar to the free field wavepacket in [27].

The central value k_0 can be determined from the classical solution $\phi \sim \frac{\lambda}{6}\alpha$ (we take $\phi = 0$ at $\alpha = 0$) by identifying the location of the “ridge” where the distribution maximizes at any slice of a given a . The result is $k_0 = -6/\sqrt{36 - \lambda^2}$. Note that in our convention $\kappa = \sqrt{6}$, $\lambda < \sqrt{2}\kappa \approx 3.46$, which is below 6 and therefore k_0 is real and finite.

FIG. 3: Distribution of the wavepacket solution (33)



D. Revised Calculation of Early-time Entanglement Entropy

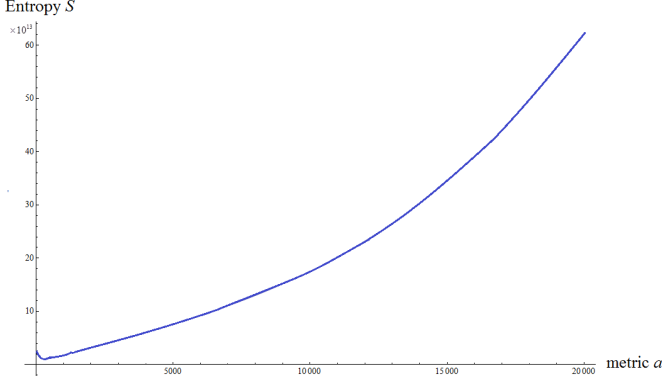
With the wavefunction distribution, Eq.(33), at hand, we can compute the von Neumann entropy $S = -\int da d\phi |\Psi|^2 \ln |\Psi|^2$. Note that if our wavepacket has perfect peak with respect to ϕ then we will recover the equation of motion (4) by saddle point approximation upon the ϕ integration, and the entropy will be the same as in section IV A.

The Resulting entropy profile is shown in figure 4 (for $\lambda = 3.3, \sigma = 0.8$).

The general tendency of the entropy evolution is accelerating, which patches well with the result in Figure 1. This is expected since we use a well-defined wavepacket.

In Figure 4, there's an early period where the numerical computation becomes less stable; however, in the

FIG. 4: Early time homogeneous entanglement entropy evolution with quantum correction



range of tolerance one can observe an entropy minimum occurs in the early period.

We give two explanations here: the entropy minimum indicates a decreasing of entropy at the onset of inflation, where the quantum fields are out of equilibrium, thus thermodynamics are not expected to hold exactly. Another possibility is minisuperspace approximation fails to hold at early time, which might be tested by adding back degrees of freedom. In our next part of inhomogeneous entropy we will pursue this phenomena further.

V. ENTANGLEMENT ENTROPY FROM COSMOLOGICAL PERTURBATIONS

In this section we will address the part of entanglement entropy induced by the entanglement between different momentum modes, which we call the “inhomogeneous” entropy. This contribution can be calculated using the standard cosmological perturbation techniques [28] [29]. The calculation in this section largely follows [17].

A. Computation of Entropy from Cosmological Perturbation

Let us denote by ν the usual Mukhanov-Sasaki variable. The comoving curvature perturbation is then $\zeta(x, t) = \nu(x, t)/z(x, t)$, with $z(x, t) = a\sqrt{\epsilon}/4G$ and

$$\epsilon = -d \ln H / d \ln a \quad (34)$$

is one of the slow-roll parameters. The conjugate momentum of ζ is denoted as $\pi = \partial_\mu \zeta$. The Gaussian random state in a single scalar field inflation is then characterized by the covariant matrix between two momentum modes \mathbf{k} and $-\mathbf{k}$, which is related to the density matrix $\bar{\rho}$ by [28]:

$$C = \text{Tr}(\bar{\rho} V V^\dagger) = \begin{pmatrix} P_\zeta & P_{\zeta\pi} \\ P_{\zeta\pi} & P_\pi \end{pmatrix} \quad (35)$$

where the trace is taken over the functional space, and

$$V = (\zeta_{\mathbf{k}}, \pi_{-\mathbf{k}}),$$

where $\pi = 2a^3\epsilon\dot{\zeta}$ is the momentum conjugate with ζ .

Each component in Eq.(35) can be identified (to tree level) as:

$$P_\zeta(\mathbf{k}, t) = |\zeta(\mathbf{k}, t)|^2, \quad (36)$$

$$P_{\zeta,\pi}(\mathbf{k}, t) = \frac{a^3\epsilon}{4\pi G} \text{Re}(\zeta(\mathbf{k}, t) \partial_t \zeta(\mathbf{k}, t)^*), \quad (37)$$

$$P_\pi(\mathbf{k}, t) = \left(\frac{a^3\epsilon}{4\pi G}\right)^2 |\partial_t \zeta(\mathbf{k}, t)|^2. \quad (38)$$

We can then evaluate the entanglement entropy from cosmological perturbation by the equations:

$$S = 2 \sum_{\mathbf{k}} [(n_{\mathbf{k}} + 1) \ln(n_{\mathbf{k}} + 1) - n_{\mathbf{k}} \ln n_{\mathbf{k}}], \quad (39)$$

$$(n_{\mathbf{k}} + \frac{1}{2})^2 = \det C_{\mathbf{k}}. \quad (40)$$

The field $\nu(\mathbf{k}, t)$ satisfies in Fourier space the equation

$$\ddot{\nu} + \frac{3\nu}{t} \dot{\nu} + \frac{k^2}{(Ft)^{2\nu}} \nu = 0, \quad (41)$$

which governs the evolution of $\zeta(\mathbf{k}, t)$. Here $F \equiv (1/(2\sqrt{3\kappa}))\lambda^2 A^{1/2(1+\gamma)}$ is a constant.

The basic steps to calculate the entropy from Eq.(39) are: (i) evolving the background metric by solving Einstein equation; (ii) solving the comoving curvature perturbation at time t for different modes \mathbf{k} according to Eq.(41); (iii) summing the entropy contributions from all modes which amounts to integrating Eq.(39) over the momentum space \mathbf{k} . In the calculation we employ the full tree-level scalar potential without early or late time approximation, given explicitly in [19].

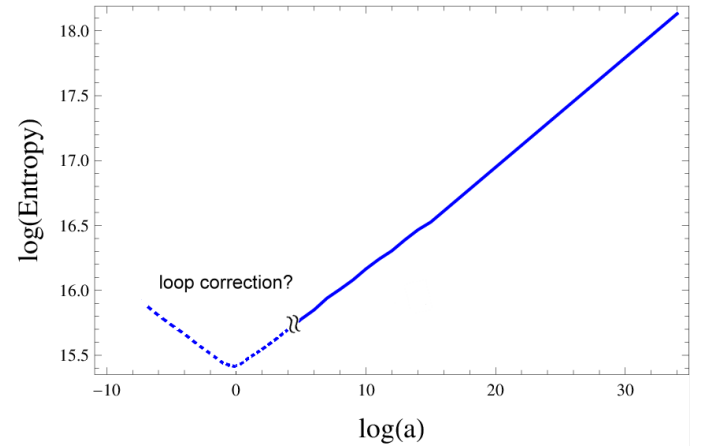


FIG. 5: Inhomogeneous entanglement entropy from tree-level cosmological perturbation

The numerical results are shown in Figure 5. Observe that the tree-level contribution increases as power law in the radiation-dominant epoch started with $S_{\text{inhom}} \sim$

10^{16} , which compensates almost entirely the exponential decrease of the homogeneous part (section IV A) of the entropy, which drops below $S_{\text{hom}} \sim 10^{12}$ after $(a/a_0) \sim 10^{13}$. On the other hand, the homogeneous entropy during early times continues to increase, resulting in the monotonic rise of the net entropy in the semiclassical regime after the decoherence of the inflaton.

In the early stage of inflation there exists an entropy minimum, which confirms the result in Fig 4 of Section IV D that even after adding back some of the truncated degrees of freedom, there must still remain a persistent entropy minimum. We can therefore expect that the entropy minimum may actually persist to higher order quantum correction, rendering this turning point a real physical feature. We include some discussions about higher order correction in Appendix B.

VI. CONCLUSION

In this paper we reconsidered inflation as a possible solution to the entropy problem on the basis neglected in the past literature that the probability of inflation, when taking its accompanied horizon into consideration, is not necessary small. Indeed our computation suggests that the initial entanglement entropy should have a sizable value in order to compensate for its subsequent reduction at the beginning of inflation (cf. Fig. 5), which indicates that the onset of inflation could be highly probable.

We explored this notion by investigating the evolution of entanglement entropy from the onset of inflation to the radiation era using a toy model, which we believe to be generic enough to capture the essential feature of entanglement entropy evolution because most calculations we made do not involve the model's artificial transition period between the two epochs, while at the same time it has the advantage of preserving unitarity instead of a discontinuous phase transition, which is disallowed in the quantum mechanical framework for the entire universe.

Our results of quantum entanglement entropy evolution are shown in Figure 1 for homogeneous early evolution after decoherence of inflaton, Figure 2 for homogeneous late-time evolution, Figure 4 for homogeneous early evolution near the onset of inflation, and Figure 5 for inhomogeneous evolution at the tree-level.

The net result for the period after decoherence of inflaton (in our case provided by the generalized Chaplygin gas at early time) is that the entanglement entropy increases monotonically, which results in a large amount of entropy generation and agrees with our experience with thermodynamic laws.

On the other hand, the situation before decoherence is more subtle. In particular, entropy minimum arisen from quantum corrections at early times appears in both the homogeneous part (Fig. 4) as well as the tree-level inhomogeneous part (Fig. 5) previously observed in [17]. This minimum may result from nonequilibrium effects during the early stage of accelerating expansion. The

persistence of the entropy minimum after putting back some of the degrees of freedom suggests that it is a real physical feature and we expect this entropy minimum not be completely erased by higher order corrections.

Our results can be summarized as follows: after the onset of inflation the entropy will soon evolve with the expanding universe dynamically to a minimum and rapidly increase hereafter. We therefore suggest that the special initial condition required for the thermodynamic arrow can be naturally provided by the inflation itself without the need to introduce any ad hoc assumptions.

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Appendix A: Range of Parameters for Inflation

In this appendix we will show it is possible in our model to realize inflation starting from an expanding but non-inflationary stage, thereby the parameter space for inflation can be wider than considered in section III B, although in this paper we restrict to the case of inflation at the start.

For the inflation to start, the kinetic energy must be small in comparison with the potential energy [7], which in our model amounts to the situation where the time derivative of the field

$$\frac{1}{2}\dot{\phi}^2 = \frac{1}{2}\frac{d\phi}{da}a^2 \sim H^2 \sim \frac{1}{2}Ca^{-(1+\beta)/(1+\gamma)} \quad (\text{A1})$$

as well as its spatial derivative, $\frac{1}{2}k^2a^{-2}\phi^2$, be small in comparison with the potential

$$V \sim \left(1 - \frac{1+\beta}{1+\gamma}\right) Ca^{-(1+\beta)/(1+\gamma)}. \quad (\text{A2})$$

The former requires

$$1 + \beta > 3(1 + \gamma), \quad (\text{A3})$$

while the later will be the case after $a \gg \exp(1/r)$ with $r = 2 - (1 + \beta)/(1 + \gamma)$. Comparing to the requirement of early inflation at $a \rightarrow 0$ in Eq.(10), we see that there is a finite nonzero range of parameters where the kinetic term is suppressed as the universe grows but not necessarily inflates, thereby paving the way for the system to become “potential dominated” and ready for the onset of inflation.

Appendix B: Higher Order Quantum Corrections to Inhomogeneous Entropy

In this appendix we briefly discuss higher order corrections to the inhomogeneous entropy. Quantum corrections will only be important at early times. According to Eq.(5), the scalar field potential has the form

$$V = V_0 e^{-\lambda\phi}, \quad (\text{B1})$$

where we have absorbed the constant ϕ into V_0 . Thus in comparison to the lowest order calculation, where only modes k and $-k$ are entangled:

$$S = \int dt \frac{d^3k}{(2\pi)^3} \epsilon \frac{1}{2} a^3 \left(\dot{\zeta}_k \dot{\zeta}_{-k} - a^{-2} k^2 \zeta_k \zeta_{-k} \right), \quad (\text{B2})$$

there is an additional entanglement due to the nonlinear interaction of Eq.(B1). This contribution can be partially accounted for by the noise kernel $N_k(t, t')$ and the diffusion kernel $D_k(t, t')$ [29]:

$$\begin{aligned} \frac{1}{2} \frac{d \det(C)}{dt} &= \left(\frac{a^3 \epsilon}{4\pi G} \right)^2 \cdot \\ &\left\{ \mathcal{P}_{\zeta\zeta}(t) \int_{-\infty}^t dt' [D_k G(t, t') - N_k G_{\text{ret}}(t, t')] \right. \\ &\left. + \mathcal{P}_{\zeta}(t) \int_{-\infty}^t dt' [N_k \partial_t G_{\text{ret}}(t, t') - D_k \partial_t G(t, t')] \right\}, \end{aligned} \quad (\text{B3})$$

with N_k and D_k obtained through the standard diagrammatic rules, where higher-order loop effect enters.

However, upon expansion of the potential Eq.(B1) one finds infinitely many types of vertexes, where each type will have its own coupling due to counter terms:

$$V_{\text{quantum}}(\phi) = c_0 + c_1 \phi + c_2 \phi^2 + \dots \quad (\text{B4})$$

This immediately leads to problems when performing perturbative loop calculations. Therefore numerical lattice computation seems desirable to obtain higher order corrections to early time inhomogeneous entropy.

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